Foundations of Mathematics and Pre-calculus 10

Module 2 Blackline Masters

This blackline master package, which includes all section assignments, as well as selected worksheets, activities, and other materials for teachers to make their own overhead transparencies or photocopies, is designed to accompany Open School BC's Foundations of Mathematics and Pre-calculus (FMP) 10 course. BC teachers, instructional designers, graphic artists, and multimedia experts developed the course and blackline masters.

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Module 2, Section 1—Lesson A: Essential Questions

Essential Questions	Before the Lesson: What I Know	After the Lesson: What I Learned	Examples
What are prime numbers, and how are they important in factoring?			
What is the GCF of two numbers, and how do you find it?			
What is the LCM of two numbers, and how do you find it?			

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Module 2, Section 1—Lesson B: Essential Questions

Essential Questions	Before the Lesson: What I Know	After the Lesson: What I Learned	Examples
How do you know if a given whole number is a perfect square or perfect cube?			
What are some strategies for solving for the square root of a perfect square or cube root of a perfect cube?			

Activity 3 Math Lab: Manipulatives

Materials

- 150 counters, pennies, or pebbles, or
- graph paper (from Appendix)
- 150 unit blocks or sugar cubes

Procedure

The following steps assume you are using pennies and sugar cubes, but you could also use counters and unit blocks respectively.

Step 1: Arrange the pennies into squares with different side lengths. Record the total number of pennies used for each square in the first table on the data sheet.

> If you are using graph paper, shade in squares with different side lengths. Record the total number of blocks shaded for each square in the first table on the data sheet.

Step 2: Arrange the sugar cubes into cubes with different side lengths. Record the total number of sugar cubes used for each cube in the second table on the data sheet.

Perfect Squares

Side Length	Area (expressed as side x side)	Area
	1 × 1	

My Notes

My Notes

Perfect Cubes

Side Length	Area (expressed as side x side x side)	Volume
	1 × 1 × 1	

Analysis

Answer the following questions:

- 1. Which numbers less than 150 are perfect squares?
- 2. Which numbers less than 150 are perfect cubes?
- 3. How could you use a sheet of graph paper, instead of counters or pennies, to do Step 1 in the procedure?
- 4. What other methods could you use to determine whether a number is a perfect square or perfect cube?



Turn to Solutions at the end of the module and mark your work. Contact your teacher if you have any questions.

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Module 2, Section 1—Lesson C: Essential Questions

Essential Questions	Before the Lesson: What I Know	After the Lesson: What I Learned	Examples
What is the difference between irrational and rational numbers?			
How can you use a graphic organizer to represent the relationship among the subsets of the real numbers (natural, whole, integer, rational, and irrational)?			
How can you quickly determine the approximate value of an irrational number given by the square roots of non-square numbers, or the cube roots of non- cubic numbers?			
What are some strategies for approximating the locations of irrational numbers on a number line?			

Section Assignment 2.1 Part 1 Factors and Multiples

- 1. Answer the following questions. Use your course materials, textbook, or the Internet to find the definition for part a). (3 marks)
 - a. What is a prime number?
 - b. According to the definition of a prime number, is 1 prime? Why or why not?
 - c. According to the definition of a prime number, is 0 prime? Why or why not?
- 2. Describe two different methods to find both the LCM and the GCF of two numbers. You may use the numbers 12 and 15 as an example. (4 marks)
 - a. GCF Method 1

b. LCM Method 1

3. Use a factor tree to find the prime factorization of 216. Show the tree and the prime factorization.(2 marks)

- 4. List the prime factorization of 357. (2 marks)
- 5. Find the GCF and the LCM of the following pairs or groups of numbers. Show your work. (12 marks; each 1 mark for work, 2 marks for answers)
 - 1. 54 and 84

GCF _____

LCM _____

2. 270 and 306

GCF _____

LCM _____

3. 65, 70 and 190

GCF _____ LCM _____

4. 900, 1260 and 2310

GCF _____ LCM _____

6. Rosa is making a game board that is 32 cm by 72 cm. She wants to use square tiles. What is the largest tile she can use? (2 marks)

7. Two bikers are riding a circular path. The first rider completes a round in 12 minutes. The second rider completes a round in 18 minutes. If they both started at the same place and time and go in the same direction, after how many minutes will they meet again at the starting point? (2 marks)

Section Assignment 2.1 Part 2 Square Roots and Cube Roots

1. Factor the following numbers and state whether they are perfect squares, perfect cubes, or neither. Show your work by grouping the factors. (6 marks)

a. 900

b. 729

c. 96

2. A cube has a volume of 2197 cu cm. What is the surface area of the cube? (3 marks)

3. Mary has 1500 square blocks. What is the largest number of blocks that she can use to construct a cube? Show all work. (2 marks)

4. What is a three-digit number that is a perfect square and perfect cube? (3 marks)

5. A litre of paint covers 8.6 square meters. How many litres would you need to purchase to paint a cube with a volume of 512 cubic meters? (3 marks)

- 6. The number 1 is both a perfect square and a perfect cube. Its square root is 1 and its cube root is also 1.
 - a. There are other numbers that are both perfect squares and perfect cubes. Can you find at least three of these numbers? List them. (1 mark)

 b. Suggest a way to find more numbers that are both perfect squares and perfect cubes. (1 mark)

Section Assignment 2.1 Part 3 Irrational Numbers

1. List at least one advantage and one disadvantage for each of the three methods used to locate an irrational number on a number line. (6 marks)

Method	Advantage	Disadvantage
Calculator		
Benchmarks		
Compass and straightedge		

2. Identify the following numbers as rational or irrational numbers and give a reason. The first one is done for you. (10 marks)

Number	Rational or Irrational?	Explanation
√ <u>10</u> = 3.16227	Irrational	non-terminating, non-repeating decimal
4∕ <mark>81</mark> = 3		
³ √81 = 4.3267		
$\sqrt{\frac{16}{9}} = \frac{4}{3}$		
√1.25 = 1.11803		
1.75		
- 18/4		
$\frac{11}{9}=1.\overline{2}$		
-3		
∛−18 = −2.6207		
√ <u>16</u> = 4		

3. Place the numbers from the table in Question 2 on the number line. Use your calculator to check that your results are accurate. (5 marks; 0.5 mark each)



Complete the following nested diagram of the real number system. Write the labels for rational numbers, irrational numbers, integers, whole numbers, and natural numbers.
(7 marks; 0.5 mark each)

Insert the following numbers into your diagram in the correct category for each:

$$0 \quad 1 \quad \pi \quad -17 \quad \frac{10}{9} \quad 2.010213... \quad \sqrt[3]{100} \quad \sqrt[4]{625} \quad -9.222$$



Section Assignment 2.1 Part 4 Glossary

Write a short definition from your personal glossary for each term below. (1 mark each; 10 marks)

- composite number
- greatest common factor (GCF)
- least common multiple (LCM)
- prime factorization
- cube root
- perfect cube
- perfect square
- square root
- irrational number
- rational number

Section Assignment 2.1 Part 4 Multiple Choice

20 marks: 2 marks each

No calculator may be used for this part of the section assignment.

1. Which of the following statements are true?

I	$\sqrt{16}$ = 8 since 8 + 8 = 16
11	$\sqrt{36}$ = 6 since 6 × 6 = 36
111	$\sqrt[3]{8}$ = 2 since 2 × 2 × 2 = 8
IV	$\sqrt[3]{16}$ = 4 since 4 × 4 = 16

- a. I and III only
- b. I and IV only
- c. II and III only
- d. II and IV only
- 2. Which of the following statements are true?

I	The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36
11	The prime factorization of 36 is $2^2 \times 3^3$.
111	$\sqrt{36}$ is an irrational number.
IV	The prime factors of 36 are 2 and 3.

- a. I and IV only
- b. II and III only
- c. II, III, and IV only

d.

I, II, III, and IV

- 3. Simplify:
 - a. $3\sqrt{6}$
 - b. $6\sqrt{3}$
 - c. $12\sqrt{3}$
 - d. $54\sqrt{3}$
- 4. What is the least common multiple of 12 and 30?
 - a. 2×3
 - b. $2^2 \times 3$
 - $c. \quad 2^3\times 3^2\times 5$
 - $d. \quad 2^2\times 3\times 5$
- 5. What is the greatest common factor of 16, 48, 80, and 96?
 - a. 2
 - b. 8
 - c. 16
 - d. 480

You may use your calculator for the last five questions.

- 6. Which two numbers have the following properties?
 - Their GCF is 15.
 - Their LCM is 225.
 - a. 3 and 5
 - b. 45 and 75
 - c. 75 and 225
 - d. 225 and 3375

7. Which of the following number lines best represents the placement of P, Q, R, given:



- 8. Simplify: $\sqrt[3]{96}$
 - a. $2\sqrt[3]{12}$
 - b. $12\sqrt[3]{2}$
 - c. $8\sqrt[3]{12}$
 - d. $8\sqrt[3]{2}$
- 9. Which one of the following sets of numbers contains only irrational numbers?

a.
$$\left\{1.78, -\frac{8}{7}, \sqrt{2}\right\}$$

b. $\left\{-3.1, \frac{2}{3}, \sqrt{16}\right\}$
c. $\left\{\sqrt{3}, \pi, \sqrt[3]{27}\right\}$
d. $\left\{\pi, 4\sqrt{2}, \sqrt[3]{10}\right\}$

10. Which of the following number lines best represents the placement of F, G, H, given:



Title	Marks
Part 1: Factors and Multiples	/27
Part 2: Square Roots and Cube Roots	/17
Part 3: Irrational Numbers	/28
Part 4: Glossary	/10
Part 5: Multiple Choice	/20
Total Marks	/102

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Module 2, Section 2—Lesson A: Essential Questions

Essential Questions	Before the Lesson: What I Know	After the Lesson: What I Learned	Examples
What is a radical?			
How do you change a mixed radical to an entire radical?			
How do you change an entire radical to a mixed radical?			
How can you order radicals on a number line?			

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Module 2, Section 2—Lesson B: Essential Questions

Essential Questions	Before the Lesson: What I Know	After the Lesson: What I Learned	Examples
What is the Zero Exponent Law?			
What is the Integral Exponent Law?			

Exponent Laws Chart

Exponent Law	General Case	Example
Product Law	$a^m \times a^n = a^m + n$	$4^3 \times 4^2$
		$=(4 \times 4 \times 4)(4 \times 4)$
		$=4 \times 4 \times 4 \times 4 \times 4$
		$=4^{5}$
Quotient Law	$a^m \div a^n = a^{m-n}, a \neq 0$	$\frac{3^6}{3^4}$
		U
		$=\frac{\cancel{3}\times\cancel{3}\times\cancel{3}\times\cancel{3}\times3\times3\times}{\cancel{3}\times\cancel{3}\times3$
		$=3\times3$
		$=3^{2}$
Power of a Power	$\left(a^{m}\right)^{n} = a^{m} \times^{n}$	$(7^2)^4$
		$= (7 \times 7)(7 \times 7)(7 \times 7)(7 \times 7)$
		$= 7 \times 7$
		$=7^{8}$
Power of a Product	$(ab)^m = a^m b^m$	$(2 \times 5)^3$
		$=(2\times5)(2\times5)(2\times5)$
		$= 2 \times 2 \times 2 \times 5 \times 5 \times 5$
		$=2^3 \times 5^3$
Power of a Quotient	$\frac{a}{b}m = \frac{a^m}{b^m}, b \neq 0$	$\frac{3}{8}^4$
		$=\frac{3}{8}\frac{3}{8}\frac{3}{8}\frac{3}{8}\frac{3}{8}$
		$=\frac{3\times3\times3\times3}{8\times8\times8\times8}$
		$=\frac{3^4}{8^4}$

Activity 2 Math Lab: Examining Patterns

Materials

- 8 pieces of shoelace licorice or French fries of equal length
- knife

Procedure

Follow the steps. Record your information in the chart:

- **Step 1:** Starting with eight licorice pieces, remove (or eat) four pieces.
- **Step 2:** In the first row of the chart, record the number of licorice pieces that remain.
- **Step 3:** Now eat half of the licorice that is left.
- **Step 4:** Repeat Steps 1 and 2 until you have one piece of licorice left.
- **Step 5:** Use the knife to cut the remaining licorice piece into two halves.
- **Step 6:** Eat one-half of the licorice and record the fraction of licorice that is left (for example, $\frac{1}{2}$, $\frac{1}{4}$, etc).
- **Step 7:** Repeat Steps 4 and 5 until it's too difficult to cut the remaining licorice in half.
- **Step 8:** Complete the second row of the chart by writing the number of licorice pieces as powers of 2.

Number of Licorice Pieces	8	4			
Expressed as Powers of 2	2 ³				

My Notes

My Notes

Analysis

Answer the following questions:

- 1. What is the pattern in the first row of the chart?
- 2. What is the pattern in the second row of the chart?
- 3. What is the value equal to 2^o? Can you give a reason for this?

4. What is the connection between the fractions in the first row and their equivalent powers of 2?

5. If you were able to continue slicing the licorice in half, why would you never be able to completely eat the licorice?



Turn to Solutions at the end of the module and mark your work. Contact your teacher if you have any questions.

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Module 2, Section 2—Lesson C: Essential Questions

Essential Questions	Before the Lesson: What I Know	After the Lesson: What I Learned	Examples
How are powers and radicals related to each other?			

Exponent Laws

Exponent Law	General Case	Example
Product Law	$a^m \times a^n = a^{m+n}$	
Quotient Law	$a^m \div a^n = a^{m-n}, a \neq 0$	
Power of a Power	$(a^m)^n = a^{m \times n}$	
Power of a Product	$(ab)^m = a^m b^m$	
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	
Zero Exponent Law	$a^{0} = 1$	
Integral Exponent Law	$a^{-m} = \frac{1}{a^m}$, where $a \neq 0$	

Discover

In this lesson you have learned about both radicals and powers. How does a power relate to a radical? You will investigate this in the following math lab.

Activity 2

Math Lab: Powers and Radicals

Materials

• scientific calculator

Procedure

Follow the steps to complete each chart to determine the relationship between a power and a radical.

Part A

Fill out the following chart. Use your calculator to evaluate the expressions in the second column $(25^{\frac{1}{2}}, \text{ for example}).$

Fill in the last row of the chart with a positive number of your choice.

x	$x^{\frac{1}{2}}$	\sqrt{x}
1		
4	$4^{\frac{1}{2}} = 2$	$\sqrt{4} = 2$
25		
576		

My Notes

My Notes

Part B

Fill out the following chart. Use your calculator to evaluate the expressions in the second column $(27^{\frac{1}{3}}, \text{ for example}).$

Fill in the last row of the chart with a positive number of your choice.

X	$x^{\frac{1}{3}}$	$\sqrt[3]{x}$
1		
8	$8^{\frac{1}{3}} = 2$	$\sqrt[3]{8} = 2$
27		
343		

Lab Analysis

1. What did you discover about the numbers in the second and third columns?

2. Why did the number you chose for the last row in Part A have to be positive? Does the number have to be positive for Part B?

3. Do you get the same results if the number you select is not a perfect square or a perfect cube?

4. What do you think the exponents $\frac{1}{4}$ and $\frac{1}{5}$ mean?

5. How would you write $47^{\frac{1}{5}}$ as a radical?



Turn to Solutions at the end of the module and mark your work. Contact your teacher if you have any questions.

My Notes

Section Assignment 2.2 Part 1 Mixed and Entire Radicals

1. Write the following entire radicals as mixed radicals. (8 marks)

a. $\sqrt{12}$ b. $\sqrt{80}$ c. $\sqrt{50}$ d. $\sqrt{108}$ e. $\sqrt[3]{16}$ f. $\sqrt[3]{135}$

g. $\sqrt[3]{81}$

h. $\sqrt[3]{448}$

- 2. Write the following mixed radicals as entire radicals. (8 marks)
 - a. $6\sqrt{2}$ b. $5\sqrt{5}$

c. $7\sqrt{3}$ d. $3\sqrt{7}$

e. $2\sqrt[3]{3}$ f. $4\sqrt[3]{3}$

g. $2\sqrt[3]{5}$

h. $5\sqrt[3]{2}$

Section Assignment 2.2 Part 2 Integral Exponents

- 1. Give an example of each of the following and explain what type of number will result. In your examples, do not use 1 or −1 for either the base or the power. The first one is done for you. (6 marks)
 - a. A negative integer raised to a negative even exponent

Example:
$$(-2)^{-2} = \frac{1}{(-2)^{-2}} = \frac{1}{4}$$

This results in a positive fraction (rational number) with a numerator equal to 1.

b. A negative integer raised to a negative odd exponent

c. A negative integer raised to a positive even exponent

d. A negative integer raised to a positive odd exponent

2. Evaluate each power without a calculator. (6 marks)



3. Simplify. Express all answers with only positive exponents. (4 marks)



4. Simplify. Express all answers with only positive exponents. (4 marks)


5. Write as a single power with positive exponents. (2 marks)



Section Assignment 2.2 Part 3 Rational Exponents

1. Evaluate without using a calculator. (3 marks)



2. Write each radical as a power. (3 marks)



3. Arrange in order from least to greatest. (4 marks)

$$\left(\frac{1}{3}\right)^{\frac{5}{2}}$$
 3² 3 ^{$\frac{5}{2}$} $\sqrt[3]{3}$

- 4. Simplify. Write your answers with positive exponents. (4 marks)
 - a. $x^{\frac{3}{2}} \cdot x^{\frac{5}{2}}$ b. $x^{-\frac{5}{2}} \div x^{\frac{1}{2}}$

c.
$$\frac{-12x^{-3}y^{\frac{3}{2}}}{4x^{2}y^{-\frac{1}{4}}}$$
 d. $\left(\frac{-27x^{9}}{y^{12}z^{-\frac{1}{3}}}\right)^{\frac{1}{3}}$

5. Simplify. Write your answers with positive exponents. (6 marks; 2 marks each)

a.
$$\frac{3m^2n^3z}{6m^{-1}n^2z^2} \cdot \left(\frac{m^3n}{2z}\right)^0$$

b. $\left(\frac{2x^2}{7}\right)^2 \cdot \left(\frac{y}{2b^3}\right)^3$
c. $\left(\frac{x^{-7}y^7}{x^{-9}y^{10}}\right)^{-3}$

Section Assignment 2.2 Part 4 Glossary

Write a short definition from your personal glossary for each term below. (1 mark each; 6 marks)

- entire radical
- index
- mixed radical
- radical
- radicand
- root

Section Assignment 2.2 Part 5 Multiple Choice

20 marks: 2 marks each

No calculator may be used for this part of the section assignment.

- **1.** Simplify: $\sqrt{96}$
 - a. $6\sqrt{4}$
 - b. $4\sqrt{6}$
 - c. $12\sqrt{6}$
 - d. $16\sqrt{6}$
- 2. Express $3\sqrt{5}$ as an entire radical.
 - a. $\sqrt{15}$
 - b. $\sqrt{45}$
 - c. √75
 - d. $\sqrt{225}$
- 3. Order the numbers from the smallest to the largest value.
 - I. $5\sqrt{3}$
 - II. $-\sqrt{49}$
 - III. $3\sqrt{7}$
 - IV. $-3\sqrt{5}$
 - a. IV, II, III, I
 - b. IV, II, I, III
 - c. II, IV, III, I
 - d. II, IV, I, III

- 4. Simplify: $\sqrt[3]{2000}$
 - a. $2\sqrt[3]{250}$
 - b. $100\sqrt[3]{20}$
 - c. $5\sqrt[3]{16}$
 - d. $10\sqrt[3]{2}$
- 5. A research assistant calculated the brain mass, *b*, of an 8 kg dog. She used the formula $b = 0.01m^{\frac{2}{3}}$, where *m* is the total mass of the dog. In which step did she make an error in her calculations?

Step I.	$b = 0.01\sqrt[3]{8^2}$
Step II.	$b=0.01\sqrt[3]{16}$
Step III.	$b\approx 0.01(2.52)$
Step IV.	$b \approx 0.025$

- a. Step I
- b. Step II
- c. Step III
- d. Step IV
- 6. Which expression is equivalent to $\frac{1}{\left(-d^4\right)^{\frac{2}{3}}}$?
 - a. $\sqrt[3]{d^8}$
 - b. $\sqrt{d^8}$
 - c. $\frac{1}{\left(\sqrt[3]{d}\right)^8}$
 - d. $\sqrt[3]{-d^8}$

7. Simplify: $\sqrt[3]{m^3n^3} \div \sqrt[3]{m^4n^6}$

a.
$$\frac{1}{mn^2}$$

b. $\sqrt[6]{m^{-1}n^{-3}}$
c. $\frac{1}{(mn^3)^{1.5}}$
d. $\sqrt{mn^2}$

You may use your calculator for the last two questions. (2 marks each)

8. Cameron wrote the following steps to evaluate without using a calculator. In which step did he make a mistake?

Step I.	$64^{-\frac{2}{3}} = \frac{1}{64^{\frac{2}{3}}}$
Step II.	$=\frac{1}{(\sqrt[3]{64})^2}$
Step III.	$=\frac{1}{4^2}$
Step IV.	$=\frac{1}{16}$

a. Step I

- b. Step II
- c. Step III
- d. He didn't make any mistakes.

9. Simplify:
$$\left[\frac{\left(a^{2}bc^{4}\right)^{2}}{a^{3}b^{3}c^{2}}\right]^{-1}$$

a. $\frac{b}{ac^{6}}$
b. $\frac{ac^{6}}{b}$
c. ac^{4}
d. $\frac{1}{ac^{4}}$

10. Evaluate and express the answer using radical notation and positive exponents.

$$x^{\frac{1}{3}} \div (xy)^{-\frac{2}{3}}$$

a.
$$\frac{\sqrt[3]{x}}{\left(\sqrt{xy}\right)^{3}}$$

b.
$$\frac{1}{\sqrt[3]{x}\left(\sqrt[3]{xy}\right)}$$

c.
$$x\left(\sqrt[3]{y}\right)^{2}$$

d.
$$\sqrt[3]{x}\left(\sqrt[3]{y}\right)^{2}$$

Title	Marks
Part 1: Mixed and Entire Radicals	/16
Part 2: Integral Exponents	/22
Part 3: Rational Exponents	/20
Part 4: Glossary	/6
Part 5: Multiple Choice	/20
Total Marks	/84

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Module 2, Section 3—Lesson A: Essential Questions

Essential Questions	Before the Lesson: What I Know	After the Lesson: What I Learned	Examples
How is multiplying binomials similar to multiplying two-digit numbers?	What I Know	What I Learned	

Tip

A way that you can remember how to multiply binomials using the distributive property is to use the acronym FOIL. The table summarizes the meaning of each letter.

Letter	Meaning of the Letter	What the Letter Means
F	First	Multiply the first terms of each binomial.
0	Outer	Multiply first term of the first binomial and the last term of the second binomial.
I	Inner	Multiply the second term of the first binomial and the first term of the second binomial.
L	Last	Multiply the last terms of each binomial.

A visual representation of FOIL is as follows.



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Module 2, Section 3—Lesson A: Essential Questions

Essential Questions	Before the Lesson: What I Know	After the Lesson: What I Learned	Examples
How do you multiply two polynomials?			
How can you verify a polynomial product?			

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Module 2, Section 3—Lesson C: Essential Questions

Essential Questions	Before the Lesson: What I Know	After the Lesson: What I Learned	Examples
How can you tell if a polynomial has a common factor in each of its terms?			
What is the relationship between multiplication and factoring of polynomials?			
How can you model the factoring of a trinomial of the form $x^2 + bx + c$?			
How can you tell if a polynomial is factored correctly?			

Activity 2 Math Lab: Exploring Factoring with Algebra Tiles

Materials

- Algebra Tiles from the FMP 10 Media CD, or
- Algebra Tiles template from the Appendix

Procedure

Follow these steps for each polynomial given.

Step 1: Select the algebra tiles which correspond to each term. For example, for the polynomial 2x + 4, you would select two *x*-tiles and four 1-tiles.



Step 2: Arrange the tiles into the shape of a rectangle. Use all of the tiles. Remember that a tile can only be placed if its sides are adjacent to other sides of equal length.



- **Step 3:** Make a sketch to show what your rectangle looks like in the chart below. (Shade the tiles to show negatives.) The first example is done for you.
- Step 4: Record the length and width of each rectangle in the chart. length: x + 2 width: 2
- **Step 5:** See if you can use the same algebra tiles to create a rectangle of different dimensions. If so, repeat steps 2–4 for the new rectangle.

Expression	Sketch of Rectangle	Length	Width	
2 <i>x</i> + 4		<i>x</i> + 2	2	
$3x^2 + 3x$				
$x^2 + 6x + 8$				
$x^2 - 5x + 6$				

Analysis

1. How can you use the dimensions of the rectangle to find the area of the rectangle?

- 2. Answer the following for the trinomials $x^2 + 6x + 8$ and $x^2 5x + 6$ and their lengths and widths from the chart above.
 - a. How is the first term of the trinomial related to the first terms in the length and width?
 - b. How is the coefficient of the middle term in the trinomial related to the second terms of the length and width?
 - c. How is the last term of the trinomial related to the second terms of the length and width?



Turn to Solutions at the end of the module and mark your work. Contact your teacher if you have any questions.

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Module 2, Section 3—Lesson D: Essential Questions

Essential Questions	Before the Lesson: What I Know	After the Lesson: What I Learned	Examples
How are the coefficients of a trinomial related to the coefficients of its binomial factors?			

Activity 2

Math Lab: Factoring $ax^2 + bx + c$

Procedure

Step 1: Multiply each of the polynomials below by using the distributive property. Write each term of the corresponding expansion in the appropriate column.

Step 2: Collect like terms and write the answer in the final column.

The first row has been done for you as an example.

	x ² -term	<i>x</i> -term	<i>x</i> -term	Constant	Product
(2x + 1)(x + 2)	2 <i>x</i> ²	+4 <i>x</i>	+1 <i>x</i>	+2	$2x^2 + 5x + 2$
(2x-3)(x+5)					
(3x+4)(2x-1)					
(3x-2)(4x-3)					

Analysis

- 1. Look at the first row containing the calculations for (2x + 1)(x + 2).
 - a. What is the product of the coefficient of the x^2 -term and the constant? (Look in the x^2 -term column and the Constant column.)
 - b. What is the product of the coefficients of the two *x*-terms? (Look in the two x^2 -term columns.)
 - c. How do the products in questions a. and b. compare?

- d. What is the sum of the two *x*-terms?
- e. What is the coefficient of the *x*-term in the final answer?
- f. How do your answers in questions a. and b. compare?
- 2. Look at the third row of the chart with the calculations for (3x + 4)(2x 1).
 - a. What is the product of the coefficient of the x^2 -term and the constant?
 - b. What is the product of the coefficients of the two *x*-terms?
 - c. How do the products in questions a. and b. compare?
 - d. What is the sum of the two *x*-terms?
 - e. What is the coefficient of the *x*-term in the final answer?

- f. How do your answers in questions d. and e. compare?
- 3. Look at the remaining rows of the chart to find the pattern, then fill in the blanks in the following statements.

Use these words: sum coefficient constant product product

The _____ of the coefficient of the x^2 -term and the

_____ is equal to the _____ of the two *x*-term

coefficients, and the ______ of the two *x*-term coefficients is

equal to the ______ of the *x*-term in the product.

- 4. If you were given the x^2 -term, the two *x*-terms, and the constant only, how could you obtain the original binomials?
- 5. Complete the table below. In each case, find a pair of numbers with the indicated product and sum. An example has been done for you.

Sum	Product	Numbers	Check
5	4	4 and 1	$4 \times 1 = 4$ 4 + 1 = 5
7	10		
1	-6		
-9	20		
_4	-12		



Turn to Solutions at the end of the module and mark your work. Contact your teacher if you have any questions.

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Module 2, Section 3—Lesson E: Essential Questions

Essential Questions	Before the Lesson: What I Know	After the Lesson: What I Learned	Examples
What is a "difference of squares"?			
How do you select appropriate strategies for factoring polynomials?			

Discover

My Notes



Photo by Martin Bílek © 2010

Activity 2 Math Lab: Investigating a Difference of Squares

Procedure

Step 1: Take graph paper and cut out a square that is 10 units by 10 units. How many square units occupy the

 10×10 square?

_	 	 _	 	 	_

- **Step 2:** In one corner of the 10×10 square, mark off a 6×6 square.
- **Step 3:** Remove the 6×6 square, and determine the area that remains.

_					_	
	 _	_				

Section Assignment 2.3 Part 1 Multiplying Binomials

- 1. Use algebra tiles to multiply the following binomials. Sketch your result, and give the final answer. (8 marks; 2 marks each)
 - a. (a-5)(a+4)

b. (2h-3)(h+2)

c. (-y + 1)(3y - 5)

d. (z + 5)(z − 2)

- 2. Multiply the following. Create a rectangle diagram to show each product. (6 marks; 2 marks each)
 - a. $(w+4)(w+3) = w^2 + 7w + 12$

b. $(2p-3)(p+6) = 2p^2 + 9p - 18$

c. $(x + y)(x + 3y) = x^2 + 4xy + 3y^2$

- 3. Multiply the following using the distributive property. (Remember FOIL if it helps.) Show all the steps in your answer. (6 marks; 2 marks each)
 - a. (p-2)(p-4)

b. (4x + 3)(x - 5)

c. (-k+4)(2k-1)

Section Assignment 2.3 Part 2 Multiplying Polynomials

Multiply the polynomials.

1.
$$(2x+5)(x^2-3x+4)$$

2.
$$(7y-1)(y^2+3y-5)$$

3.
$$(3a+2b)(-7a+5b-2)$$

4. (2p-q)(2p-4q+1)

- 5. Shonti multiplied 3x 2 by $-3x^2 + 4x 3$ and got the result $9x^3 6x^2 17x + 6$ but she thinks she made an error.
 - a. Use the left side/right side method and x = 2 to verify her answer. (2 marks)

b. Show the correct result below and verify your result using x = 2. (2 marks)

Section Assignment 2.3 Part 3 Common Factors

- 1. Factor the following: (9 marks)
 - a. 12*m*²*n* 15*m*
 - b. $22p^3 + 4$
 - c. $-5 30b^2 + 40b^3$
 - d. $x^2 + 5x 14$
 - e. $b^2 8b + 15$
 - f. $h^2 + 11h + 30$
 - g. $2y^2 + 10y 72$

h. $3c^2 + 9c - 54$

- i. $m^3n 5m^2n 24mn$
- 2. The following factorizations contain errors. Identify the error, and then provide the correct solution. (4 marks; 2 marks each)
 - a. $32x^4 + 28x^2 8x = 8x(4x^3 + 7x 1)$

Error:

Correct solution:

b. $b^2c^2 - 3bc^2 - 40c^2 = c^2(b^2 + 3b - 40)$ = $c^2(b + 8)(b - 5)$

Error:

Correct solution:

3. The following sets of algebra tiles represent the multiplication of two binomials. Determine the two binomials. (2 marks)





Section Assignment 2.3 Part 4 Factoring Trinomials

- 1. Factor completely. (8 marks; 2 marks each)
 - a. $3x^2 5x 12$

b. $2x^2 + 9x - 68$

c. $7x^2 + 42x + 35$

d. $25x^2 - 30x + 9$

- 2. Which of the following trinomials can be factored? If the trinomial can be factored, state the factors. If it cannot, explain why not. (14 marks)
 - a. $x^2 + 2x 35$

b. $x^2 - 6x - 27$

c. $x^2 - 3x - 5$

d. $2x^2 + 3x - 2$

e. $2x^2 + 9x + 4$

f. $2x^2 + 9x - 35$

g. $2x^2 - 13x + 25$

Section Assignment 2.3 Part 5 Factoring Special Trinomials

- 1. Factor the following binomials and trinomials. (4 marks)
 - a. $9x^2 + 24x + 16$

b. $4y^2 - 81x^2$

c. $8g^2 - 56g + 98$

d. 16*k*⁴−81

- 2. Factor the following. These questions are mixed, so you will have to use all of the factoring strategies that you have learned in this section. (10 marks)
 - a. $8z^2 30z 27$
 - b. $3g^{3}h + 21g^{2}h + 30gh$
 - c. $3x^3 3xy^2$
 - d. $4x^2 + 4x + 1$
 - e. $50z^3 40z^2 + 8z$
f. $9a^2 - 4$

g. $x^2 - 2x - 24$

- h. $8a^2b 20a^2c 16a$
- i. $-2x^2 19xy 24y^2$
- j. $6g^3h + 21g^3h^2 15g^2h$
- 3. Fill in the following factoring strategies chart. A question of each type has been provided in the worked example column, but feel free to substitute your own if you like. The first row has been done for you. (12 marks)

Strategy Name	When to Use	Worked Example	Tips, Hints, or Points to Remember
Greatest Common Factor	Use when the terms in a polynomial have common factors	$10x^2y + 15x^3y^2$	Whenever you have to factor, you should check first to see if this
	other than 1.	$10x^2y = (2)(5)(x)(x)(y)$	method can be used.
		$15x^{3}y^{2} = (3)(5)(x)(x)(x)(y)(y)$ GCF = (5)(x)(x)(y)	Remember to check the final expression to see if any it can be
		$= 5x^{2}y$ $10x^{2}y + 15x^{3}y^{2} = 5x^{2}y(2 + 3xy)$	factored further.
Difference of Squares	This method is used when the polynomial: 1.	$x^{2} - 4$	
	2.		
	З.		
	4.		
Grouping		$6x^2 + 2y + 15x^2y + 5y^2$	

Decomposition	Product-Sum
$2x^2 + 7x - 15$	$x^2 + 8x + 15$

	$3x^2 - 8x + 4$	
Inspection	6k ² - 11k - 35	

Section Assignment 2.3 Part 6 Multiple Choice

20 marks: 2 marks each

No calculator may be used for this part of the section assignment.

1. Which of the following diagrams best represents the expansion of (x + 2)(x + 3) pictorially?



- 2. Katie simplified the expression (x + b)(x + c), where b < 0 and c < 0, to the form $x^2 + gx + k$. What must be true about g and k?
 - a. g < 0 and k > 0
 - b. *g* < 0 and *k* < 0
 - c. *g* > 0 and *k* > 0
 - d. g > 0 and k < 0
- 3. Which of the following expressions have a factor of x 3?

Ι.	$x^2 + 6x + 9$
II.	x ² - 9
III.	$2x^2 - 3x - 9$

- a. II only
- b. II and III only
- c. III only
- d. I, II, and III
- 4. Given that the area of the rectangle below is $2x^2 x 15$, determine the length of the rectangle.



- a. 2x + 5
- b. 2x 5
- c. $2x^2 18$
- d. $2x^2 2x 12$

- 5. Factor $y^2 16$.
 - a. $(y-4)^2$ b. $(y+4)^2$ c. (y+4)(y-4)d. (y+2)(y-2)(y+4)

You may use your calculator for the last five questions. (2 marks each)

6. Kevin expanded and simplified $(x - 4)(x^2 - 2x + 3)$ as shown below.

	Steps							
Ι	$x(x^2-2x+3)-4(x^2-2x+3)$							
II	$x^3 - 2x^2 + 3x - 4x^2 - 8x - 12$							
111	$x^3 - 6x^2 - 5x - 12$							

In which step is Kevin's first error?

- a. Step I
- b. Step II
- c. Step III
- d. There is no mistake.
- 7. Determine an expression to represent the shaded area below.



- a. $4x^2 + 10x + 20$
- b. $3x^2 + 17x + 10$
- c. $2x^2 + 24x + 20$
- d. $2x^2 + 24x$

- 8. Determine the greatest common factor of $9a^5b$, $18a^3b^3$, and $24a^2b^5$.
 - a. *a*³*b*
 - b. 3*ab*
 - c. 3*a*⁵*b*⁵
 - d. 3*a*²*b*
- 9. When completely factored, how many factors does $2x^3y 18xy$ have?
 - a. 2
 - b. 3
 - c. 4
 - d. 5
- 10. Joe was asked to factor $4x^2 + 4x 15$ and represent it with math tiles.



What additional tiles would he need to represent the total area when the two factors are multiplied together?



Title	Marks
Part 1: Multiplying Binomials	/20
Part 2: Multiplying Polynomials	/12
Part 3: Common Factors	/15
Part 4: Factoring Trinomials	/22
Part 5: Factoring Special Trinomials	/26
Part 6: Multiple Choice	/20
Total Marks	/115

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