FOUNDATIONS OF MATHEMATICS 11

Introduction Assignment

Welcome to FOM 11! This assignment will help you review some topics from a previous math course and introduce you to some of the topics that you'll be studying this year. The last part of this assignment asks you to provide your teacher with information about your previous experiences in math.

In order to earn full marks for each question, you must show all your work. Where a numerical response is required, answer to the nearest tenth

Student Name _____

Student No. _____ **Date** _____

Address _____ **Postal Code** _____

_____

Complete the following *Foundations of Mathematics 11* Assignment independently and return it to your teacher based on the instructions provided by your school. No external resources are required to complete this assignment.

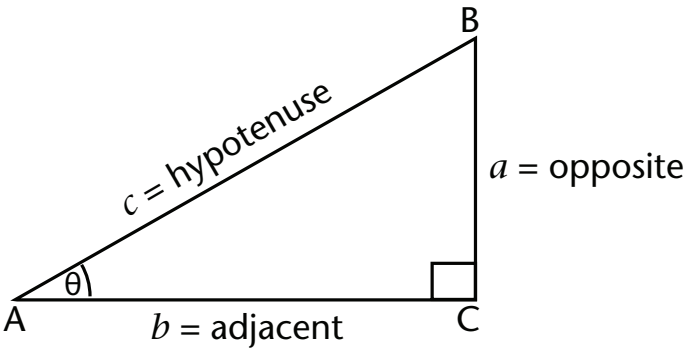
Title	Marks
Part 1: Trigonometry	/10
Part 2: Systems of Equations	/10
Part 3: Factoring Trinomials	/10
Part 4: Quadratics	/15
Part 5: About You	/5
Total marks	/50

Contents:
24 pages

Assignment time:
3 hours

Part 1: Trigonometry (10 marks)

Triangle ABC with reference angle can be labelled with the following:



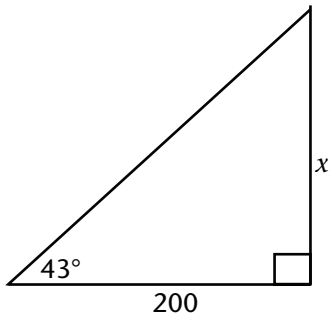
The following table summarizes the three trigonometric ratios that you learned in a previous math course.

Ratio Name	Description	Calculation	Mnemonic*
Sine	The ratio of the length of the side opposite the reference angle to the length of the hypotenuse.	$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$	S O H
Cosine	The ratio of the length of the side adjacent to the reference angle to the length of the hypotenuse.	$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	C A H
Tangent	The ratio of the length of the side opposite the reference angle to the length of the side adjacent to the reference angle.	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$	T O A

*A way to remember

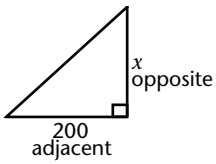
Example 1

Determine the measure of length x to the nearest tenth.



Solution

Label the triangle.



Given: one angle and the adjacent side

Need: the opposite side

Solve with tangent. SOH CAH TOA

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 43° = \frac{x}{200}$$

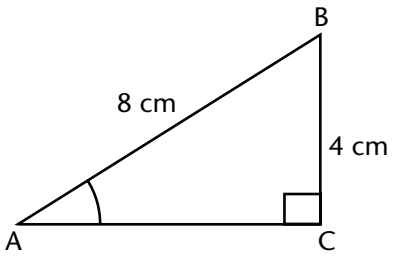
$$(200)(\tan 43°) = \left(\frac{x}{200}\right)(200)$$

$$(200)\tan(43°) = x$$

$$x = 186.5$$

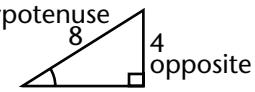
Example 2

Determine the measure of angle A to the nearest degree.



Solution

Label the triangle.



Given: the side opposite and the hypotenuse

Need: angle A

Solve with sine. (SOH) CAH TOA

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{4}{8}$$

$$\sin A = 0.5$$

$$\angle A = \sin^{-1}(\sin A) = \sin^{-1}(0.5)$$

$$\angle A = 30°$$

To "undo" sin A and discover the measure of ∠A, find the $\sin^{-1}$ of both sides of the equation.

Solving a triangle involves finding all the remaining measurements of a triangle when you're given the measure of one length and one acute angle, or the measures of two lengths. Two things to remember:

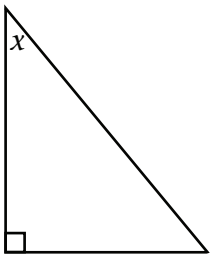
- the sum of the interior angles of a triangle is 180°, so if you're given the measure of one angle besides the right angle, you can calculate the measure of the third angle using addition and subtraction.
- you can solve for the third side of a right triangle using the Pythagorean Theorem, $a^2 + b^2 = c^2$.

MARKS

Now it's your turn.

1. Label the following triangle PQR with respect to the reference angle, x, where the right angle is at Q (there are two different ways to label it; choose one).

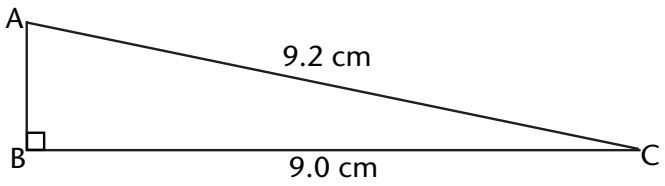
   Include the following labels: P, Q, R, p, q, r, opposite, adjacent, hypotenuse. (3 marks)



MARKS

2. Solve the following right triangles. Answer to the nearest tenth. Include units with your answers. (7 marks; 4 marks for a and 3 marks for b)

a.

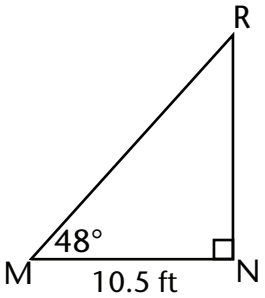


$\angle A =$ _____

$\angle C =$ _____

AB = _____

b.



$\angle R =$ _____

RN = _____

MR = _____

Part 2: Systems of Equations (10 marks)

In FOM 11 you'll be studying systems of linear inequalities. To prepare for this, following is a brief review of linear equations and systems.

Linear equations can appear in various forms. Following is an example of the same linear equation arranged in the different forms.

- standard form $2x - 3y = 4$
- point-slope form $y - 2 = \frac{2}{3}(x - 5)$
- general form $2x - 3y - 4 = 0$
- slope-intercept form $y = \frac{2}{3}x - \frac{4}{3}$

An easy way to graph a linear equation is to arrange it in slope-intercept form, written as $y = mx + b$. In the equation, m is the slope and b is the y-intercept.

Example 1
A linear function is described by the equation $4x - 3y + 12 = 0$. Identify the slope and y-intercept of the graph, and then sketch the graph.

Solution
You can rearrange the equation into slope-intercept form in order to identify the slope and y-intercept.

$$\begin{aligned} 4x - 3y + 12 &= 0 \\ -3y &= -4x - 12 \\ \frac{-3y}{-3} &= \frac{-4x}{-3} - \frac{12}{-3} \\ y &= \frac{4}{3}x + 4 \end{aligned}$$

The slope is $\frac{4}{3}$ and the y-intercept is 4.

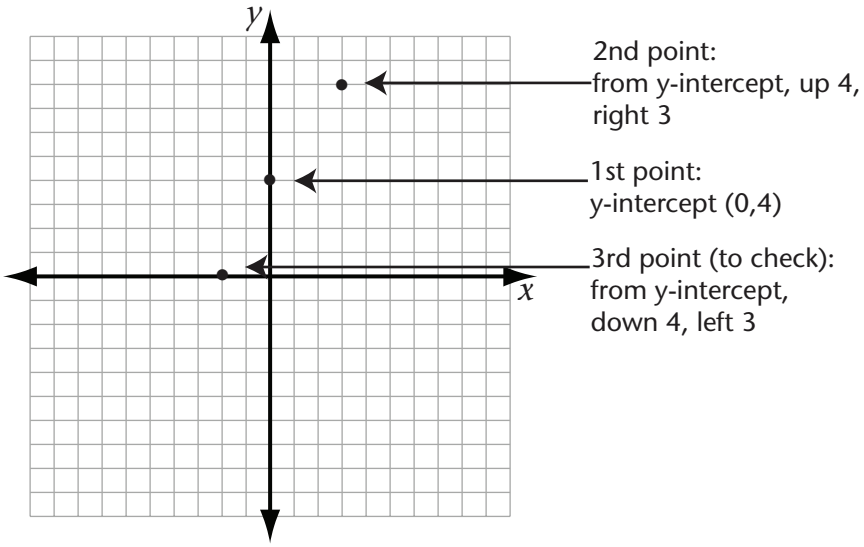
Once you have determined the slope and the y-intercept, you can then sketch a graph of the linear function.

First plot the y-intercept at (0, 4). Then apply the slope to find another point.

Since $\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{4}{3}$, count up four spaces and right three spaces to find the second point.

Remember: The positive y-direction is up, and the positive x-direction is to the right. Since $\frac{\text{rise}}{\text{run}} = \frac{4}{3} = \frac{-4}{-3}$, you could also count four spaces down and three spaces left to find another point.

We'll do this now to find a third point to check our line.



The third point checks out, so this is an accurate graph of $4x - 3y + 12 = 0$.

Systems of Linear Equations

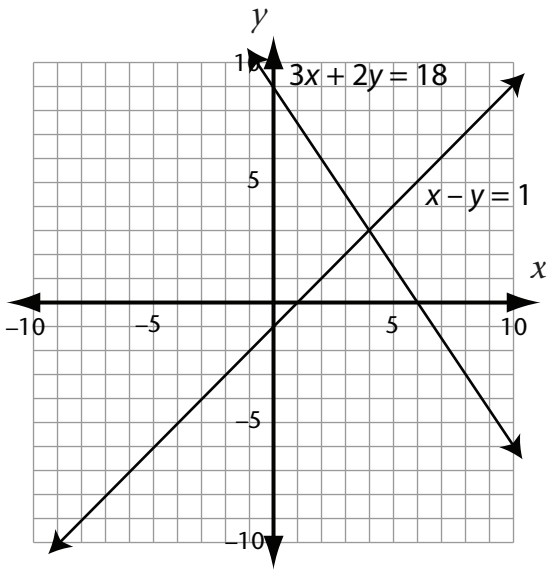
To find the solution to a system of linear equations by graphing, you must graph both lines and find the point where the lines cross. This intersection is the solution to the system of equations.

Example 2
Solve this system of linear equations by graphing.

$x - y = 1$
$3x + 2y = 18$

Solution
Graph both lines. Here we've put both equations in slope-intercept form.

$y = x - 1$
$y = -\frac{3}{2}x + 9$



The solution is found at the intersection of the graphed lines: (4, 3).

MARKS

Now it's your turn.

1. Put the following linear equations into slope-intercept ($y = mx + b$) form. (2 marks; 1 mark each)

   a. $x + 2y - 6 = 0$

   b. $15x - 3y = 2$

2. Graph each linear equation. For the first graph, use a table of values OR explain how you graphed the equation. (4 marks; 2 marks each)

   a. $2x - y = 3$

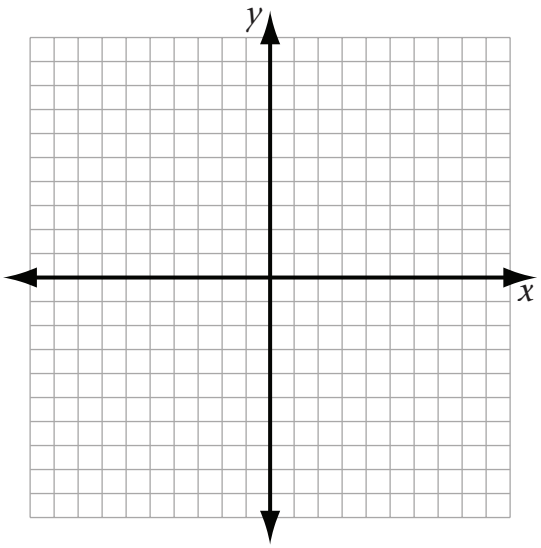


Table of Values

x	*y*

MARKS

b. $y = -\frac{2}{3}x + 1$

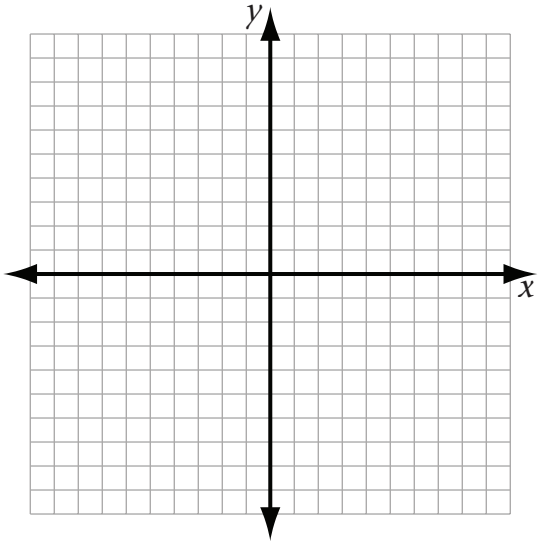
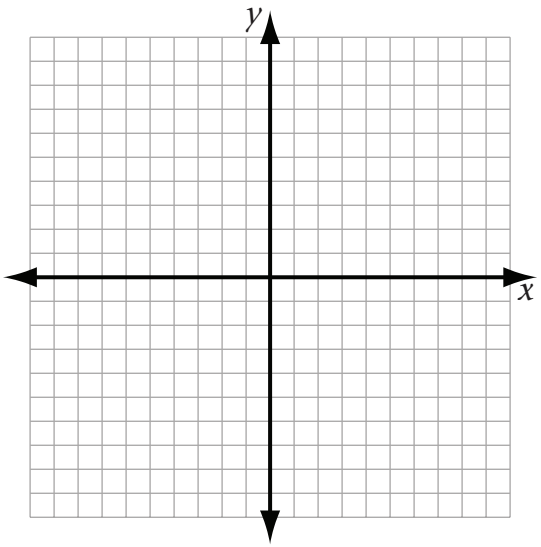


Table of Values

x	y

3. Find the solution to this system of linear equations by graphing. Show all your work. (4 marks)

   $x + y = 2$
   $4x - 3y = 15$



Part 3: Factoring Trinomials (10 marks)

You can factor trinomials of the form $ax^2 + bx + c$ by decomposition, using the tic-tac-toe method, or by inspection. Following are two examples showing the decomposition and tic-tac-toe methods.

Example 1
Factor $2x^2 + 7x - 15$ by decomposition.

Solution
Find two numbers whose product is equal to the product of (2)(–15), which is –30, and whose sum is equal to +7, the middle or b term.

By using a table or doing it in your head, you find that those numbers are +10 and –3.

$$(+10)(-3) = -30$$
$$+10 + (-3) = +7$$

Rewrite the trinomial as a four-term polynomial using the two numbers just found as the coefficients of the middle two terms.

$$2x^2 + 10x - 3x - 15$$

Group the first two and the last two terms.

$$(2x^2 + 10x) + (-3x - 15)$$

Factor out the common factor in each group.

$$2x(x + 5) - 3(x + 5)$$

Factor out the common binomial.

$$(2x - 3)(x + 5)$$

Example 2
Factor $3x^2 - 8x + 4$ using the tic-tac-toe method.

Solution
Find two numbers whose product is (3)(4), which is 12, and whose sum is –8. These numbers are –6 and –2.

Construct a 3 × 3 array as shown. Place the x^2-term of the trinomial in the centre square and the constant in the last square in the bottom row.

	$3x^2$	
		+4

Place an x-term in the last square in the middle row, and an x-term in the second square of the last row. These x-terms should have coefficients equal to the numbers determined earlier.

	$3x^2$	$-6x$
	$-2x$	+4

Now find the Greatest Common Factor (GCF) of each row and column and write them in the row and column headings. For now, make them all positive.

GCF of $3x^2$ and $-2x$ is x. GCF of $-6x$ and 4 is 2.

		x	2
GCF of $3x^2$ and $-6x$ is $3x$.	$3x$	$3x^2$	$-6x$
GCF of $-2x$ and 3 is 2.	2	$-2x$	+4

Now we'll check the sign of each GCF. $3x^2$ is positive, so x and $3x$ can stay positive. However, the product of $3x$ and the other factor (currently +2)

must be $-6x$, so we'll make the 2 negative. In the same way, the product of x and the other factor must be $-2x$, so we'll make the other 2 negative as well. Our final chart looks like this:

	x	-2
$3x$	$3x^2$	$-6x$
-2	$-2x$	$+4$

Now you can read the two binomial factors from the top row and the left-most column of the grid.

$$(x - 2)(3x - 2) = 3x^2 - 8x + 4$$

When factoring trinomials, you may also see a special case called the difference of squares. It's like a trinomial where the middle term has a coefficient of zero. Here are some examples.

$$x^2 - 49 \qquad 4x^2 - 25 \qquad 16m^2 - 81$$

To factor a difference of squares, take the square root of each term. Then write one factor as the sum of the square roots and the other factor as the difference of the square roots.

Example 3

Factor $9x^2 - 4y^2$ completely and check by multiplying.

Solution

Step 1: First, check the terms for common factors. Since there are none, continue by determining the square roots of $9x^2$ and $4y^2$.

$$\sqrt{9x^2} = 3x$$

$$\sqrt{4y^2} = 2y$$

Step 2: Write one factor as the sum of the square roots. Write the other factors as the difference of the square roots.

$$9x^2 - 4y^2 = (3x + 2y)(3x - 2y)$$

MARKS

Step 3: Check by multiplying.

$$\begin{aligned}&(3x+2y)(3x-2y)\\&=3x(3x-2y)+2y(3x-2y)\\&=9x^2-6xy+6xy-4y^2\\&=9x^2-4y^2\end{aligned}$$

The original difference of squares is recovered, so the factors are correct.

As the example above suggests, before you start factoring by decomposition or using the tic-tac-toe method, check for a common factor in each term.

Now it's your turn.

1. Factor the following. (10 marks; 1 mark each)

$x^2 + x - 6$

$2x^2 + 11x + 12$

$5x^2 - 7x + 2$

$2x^2 + 7x + 6$

$6x^2 + 11x - 10$

$x^2 + 6x + 9$

MARKS

$2x^2 - 13x + 6$

$x^2 - 6x + 8$

$4x^2 + 12x + 9$

$81x^2 - 144y^2$

MARKS

Part 4: Quadratics (15 marks)

In FOM 11 you'll be studying quadratic functions and equations. This introduction will help you review your graphing and analytical skills.

The vertex form of a quadratic function is

$$f(x) = a(x - p)^2 + q$$

where a, p, and q are constants and $a \neq 0$

Here are two examples of quadratic equations.

$y = -2(x - 1)^2 + 4$ $\qquad$ $y = \frac{1}{2}(x + 2)^2 - 1$

1. Create a table of values for each equation by substituting various values of x and calculating the corresponding y-values. Use the x-values given in each table. (4 marks)

   Note: Show your work for the first calculation in each table, but after that you can do further calculations on scrap paper, on your calculator, or in your head, if you want.

   a. $y = -2(x - 1)^2 + 4$

x	y
1	
0	
2	
–1	
3	–4
–2	
4	

MARKS

b. $y = \frac{1}{2}(x + 2)^2 - 1$

x	y
–2	
–1	–½
–3	–½
0	
–4	
1	3½
–5	
2	
–6	

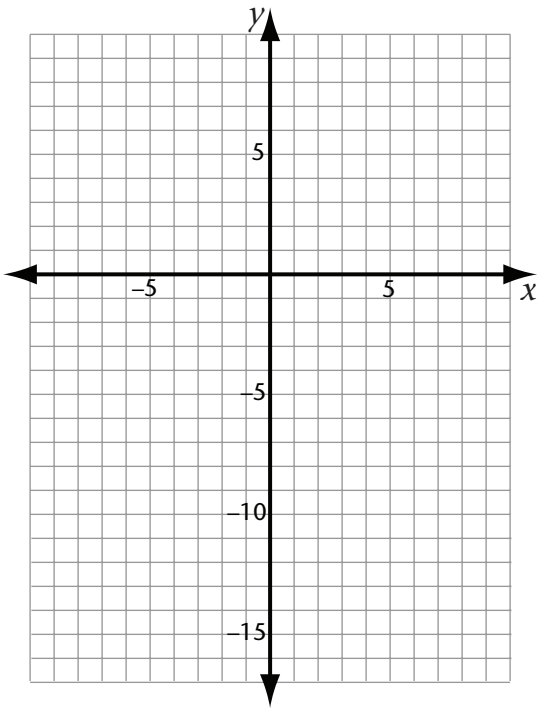
MARKS

2. Using the tables of values from above, sketch both of the graphs. (5 marks)

   Note the following:

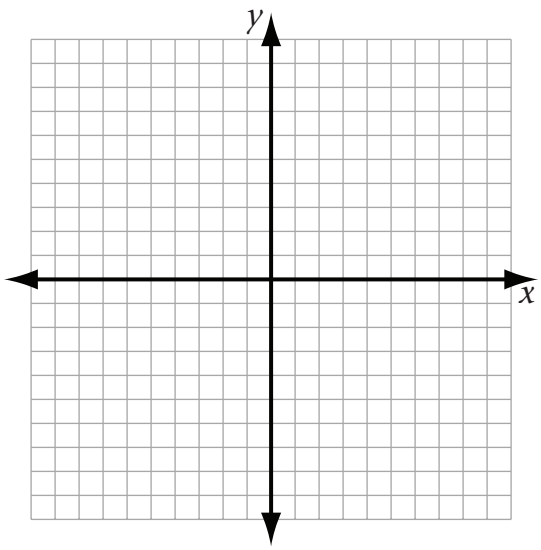
   - The graphs are continuous, so join the points with lines.
   - The graphs continue past the points we're graphing, so put arrows on the ends of the lines.

   a. $y = -2(x - 1)^2 + 4$



MARKS

b. $y = \frac{1}{2}(x + 2)^2 - 1$



3. Analyze the graphs by answering the following. Write at least two points for each question. (4 marks)

   a. What is similar in both graphs?

   _____

   _____

   _____

   b. What is different about the graphs?

   _____

   _____

   _____

MARKS

4. Each graph above has a vertex, the lowest or highest point on the graph (depending on whether it "opens upward" or "opens downward").

   What is the vertex for each graph? (2 marks)

   $y = -2(x - 1)^2 + 4$ _____

   $y = \frac{1}{2}(x + 2)^2 - 1$ _____

Part 5: About You (5 marks)

Answer the following questions about your last math course.

What was the name of your last math course? _____

_____

Explain the format of the course (e.g., in a classroom, online, print-based distance learning, etc.)

_____

_____

When did you finish it? _____

How long did it take you to complete the course? _____

_____

Did you do well in it? _____

Explain: _____

_____

What did you like best about your last math course? _____

_____

_____

Are there certain topics that you find challenging or difficult?

_____

_____

_____

What do expect to achieve in FOM 11?

Please add anything else about yourself or your previous experiences in math that may help your teacher guide you through this course.

/10	Part 1: Linear Equations and Graphing
/10	Part 2: Systems of Equations
/10	Part 3: Factoring Trinomials
/15	Part 4: Quadratics
/5	Part 5: About You
50	**Total**